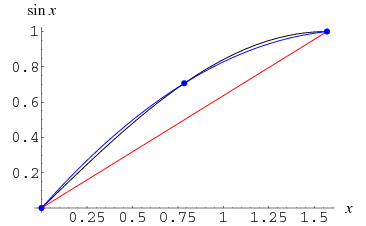
Simpson's rule is a method for numerical integration, the numerical approximation of definite integrals. Specifically, it is the following approximation for  n+1equally spaced subdivisions (where  n is even): (General Form)

Simpson's rule is a Newton-Cotes formula for approximating the integral of a function f using quadratic polynomials (i.e., parabolic arcs instead of the straight line segments used in the trapezoidal rule). Simpson's rule can be derived by integrating a third-order Lagrange interpolating polynomial fit to the function at three equally spaced points. In particular, let the function f be tabulated at points x_0, x_1, and x_2 equally spaced by distance h, and denote f_n=f(x_n). Then Simpson's rule states that

|  |  |  |  |
| --- | --- | --- | --- |
| int_(x_0)^(x_2)f(x)dx | = | int_(x_0)^(x_0+2h)f(x)dx | (1) |
| http://mathworld.wolfram.com/images/equations/SimpsonsRule/Inline11.gif | approx | 1/3h(f_0+4f_1+f_2). | (2) |

Since it uses quadratic polynomials to approximate functions, Simpson's rule actually gives exact results when approximating integrals of polynomials up to cubic degree.



For example, consider f(x)=sinx (black curve) on the interval [0,pi/2], so that f(x_0=0)=0, f(x_1=pi/4)=1/sqrt(2), and f(x_2=pi/2)=1. Then Simpson's rule (which corresponds to the area under the blue curve obtained from the third-order interpolating polynomial) gives

|  |  |  |  |
| --- | --- | --- | --- |
| int_0^(pi/2)sinxdx | approx | 1/3(1/4pi)(0+4/sqrt(2)+1) | (3) |
| http://mathworld.wolfram.com/images/equations/SimpsonsRule/Inline22.gif | = | 1/(12)(1+2sqrt(2))pi | (4) |
| http://mathworld.wolfram.com/images/equations/SimpsonsRule/Inline25.gif | approx | 1.00228, | (5) |

whereas the trapezoidal rule (area under the red curve) gives pi/4 approx 0.785398 and the actual answer is 1.

In exact form,

|  |  |  |  |
| --- | --- | --- | --- |
| int_(x_0)^(x_2)f(x)dx | = | 1/3h(f_0+4f_1+f_2)+1/6int_(x_0)^(x_1)(x_0-t)^2(x_1-t)f^((3))(t)dt+1/6int_(x_1)^(x_2)(x_2-t)^2(x_1-t)f^((3))(t)dt | (6) |
| http://mathworld.wolfram.com/images/equations/SimpsonsRule/Inline32.gif | = | 1/3h(f_0+4f_1+f_2)+R_n, | (7) |

where the remainder term can be written as

|  |  |
| --- | --- |
| R_n=1/(90)h^5f^((4))(x^*), | (8) |

with x^* being some value of x in the interval [x_0,x_2].

An extended version of the rule can be written for f(x) tabulated at x_0, x_1, ..., x_(2n) as

|  |  |
| --- | --- |
| int_(x_0)^(x_(2n))f(x)dx=1/3h[f_0+4(f_1+f_3+...+f_(2n-1))   +2(f_2+f_4+...+f_(2n-2))+f_(2n)]-R_n, | (9) |

where the remainder term is

|  |  |
| --- | --- |
| R_n=(nh^5)/(90)f^((4))(x^*) | (10) |

for some x^* in [x_0,x_(2n)].